Correcting Distorted Objects Formed in a Concave Mirror

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ABSTRACT
Concave mirrors are capable of gathering light rays and optically forming objects in midair so we may reach them by hand. In this study, we propose a concave mirror-based display technique that features large viewing angles. Generally, optical systems of concave mirrors cause formed objects to be distorted when allowing large viewing angles. We aim to correct the distortion of the formed objects by pre-warping the original ones, so that the observer sees them not-distorted. Finally we confirm our approach by both computer simulation and a simple experiment.

ACM Classification: H.5.2Information Interfaces And PresentationUser Interfaces[Interaction styles, Theory and methods]

General terms: Design

Keywords: object distortion, concave mirrors, volumetric displays.

INTRODUCTION
Head-mounted displays are widely used to enable users to perceive volumetric objects. These displays create any desired objects by displaying a pair of two slightly different images for each eye. They however cause the conflict between convergence and accommodation, which leads to eye fatigue. In order to make this conflict less severe, various display techniques have been studied to form volumetric objects by illuminating each point in a three-dimensional space. Favalora et al. used a small spinning screen where the slice of a volumetric object was projected[3]. Palovuori et al. used fog as a large projection screen[5]. Chekhovski et al. formed a volumetric object by letting a laser break down at each point in water and emit light[1]. Eitoku et al. projected images onto falling drops of water[2]. Miyazaki et al. used two concave mirrors to optically form a volumetric object by gathering light rays[4]. Table 1 summarizes the characteristics of the previous display techniques. "Reachability" means that the formed objects can be reached by hand. Eitoku et al.'s display[2] is marked "Possible" because it is not suitable for reaching them due to use of water. This study focuses on a concave mirror-based display technique that forms volumetric objects and supports both reachability and large viewing angles. Miyazaki et al's display[4] does not support large viewing angles. This is because large viewing angles cause object distortion. How could our display technique support large viewing angles? We take an approach to modeling the distortion of formed objects, and then pre-warping the original ones before they are exposed to the concave mirror. Thus, these objects form not-distorted in the mirror.

Table 1: Comparison between display techniques

Display techniques	Reach -ability	Viewing angle(deg.)	Formed data(dim.)
Spinning screen[3]	No	360	3
FogScreen[5]	Yes	360	2
Laser[1]	No	360	3
Drops of water[2]	Possible	360	3
Concave mirrors[4]	Yes	Limited	3

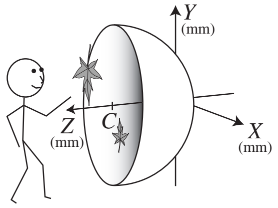

Figure 1: The look of our concave mirror-based display.

MODELING DISTORTION
Concave mirrors are capable of optically forming objects in midair. But these formed objects are affected by distortion. That is, the further an object goes away from the center, the more badly the object is distorted. In this study, we model the distortion mathematically. Figure 1 shows a schematic diagram of our concave mirror-based display. C is the center of a sphere a half part of which is the concave mirror. The radius of the concave mirror is 500mm and C is placed at $(0, 0, 500)$. The lower leaf shows an object while the upper one does the formed object that is supposed to be seen by the observer. In Figure 2, 41×21 samples of light sources are placed and they are shown in the lower group of dots. Then for each of the light sources, the corresponding conjugate point is calculated with ray tracing in a computer model of the concave mirror. The upper group of dots is the calculation result of the conjugate points. The curved arrow shows the mapping of a light source in the bottom left corner to the corresponding conjugate point. As mentioned, these conjugate points are distorted towards the concave mirror. This distortion is known dependent on an observer's eye position. This calculation result is based on the left eye position of $(-30, 50, 1000)$ and the right one of $(30, 50, 1000)$. Let (x, y, z) and (x', y', z') be the positions of light sources

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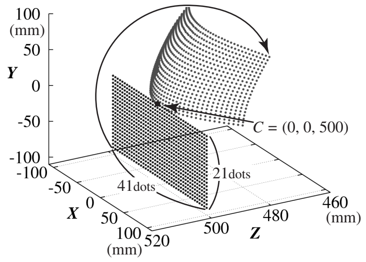

Figure 2: Light sources (the lower group of dots) and the calculated conjugate points (the upper group of dots).

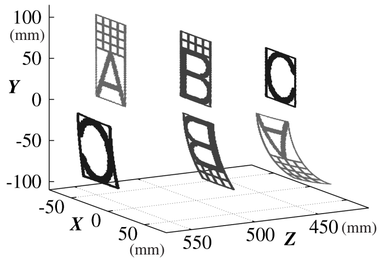

Figure 3: Three letters A, B and C (drawn above $Y = 0$) and the pre-warped letters (drawn below $Y = 0$).

and the conjugate points, respectively. Using the samples of dots in Figure 2 plus multiple layers of dots in z axis, we heuristically modeled the mapping of (x, y, z) to (x', y', z') as follows.

$$x' = -\frac{f_{r'}(r,z)}{r} \times x \quad (1) \qquad y' = -\frac{f_{r'}(r,z)}{r} \times y \quad (2)$$

$$z' = f_h(z) - f_g(z) \times \left(x'^2 + y'^2\right) \quad (3)$$

$$r = \sqrt{x^2 + y^2} \quad (4)$$

$$f_{r'}(r,z) = f_{r'}(r,500) \times \frac{f_{r'}(100,z)}{f_{r'}(100,500)} \quad (5)$$

$$f_{r'}(r,500) = 4.2891 \times \left(1 - \frac{1}{0.9838^r}\right) + 1.0789 \times r \quad (6)$$

$$f_{r'}(100,z) = \frac{43462.3851}{z - 107.7721} - 20.3874 \quad (7)$$

$$f_h(z) = \frac{64178.5596}{z - 246.4681} + 246.8226 \quad (8)$$

$$f_g(z) = \frac{0.6002}{z - 326.4267} - 0.0017 \quad (9)$$

SIMULATION

We pre-warped three letters of A, B and C using the mapping. Figure 3 shows both the three letters and pre-warped ones. The three letters are drawn above $Y = 0$ and they are pre-warped below $Y = 0$. Figure 4 shows two pairs of the left and right eye views obtained in a computer simulation of the concave mirror. The top pair is the result when the letters were pre-warped. For comparison, the bottom pair is the result when the letters were not pre-warped. The bottom pair is distorted while the top pair is not entirely distorted but the upper part is partially bent towards the mirror. The mapping fairly corrects the distortion but not complete yet. This is not only a matter of the number of samples but also the selection of equations used to model the mapping. Furthermore we performed a simple experiment in a real environment. We used an acrylic concave mirror of 30cm in diameter and placed beads manually after pre-warping their positions so that these beads would look aligned in a horizontally straight line and parallel to the mirror. The observed beads were aligned reasonably compared to those not pre-warped. This is consistent with the simulation results.

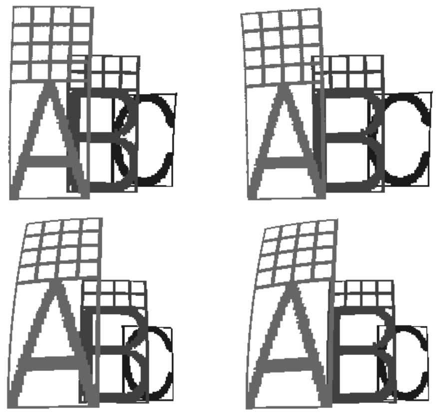

Figure 4: Simulation results. The top pair was obtained when the letters were pre-warped, while the bottom pair was obtained when the letters were not pre-warped. Each pair needs to be viewed in parallel view method.

CONCLUSIONS

Concave mirrors are capable of forming objects in midair. When allowing large viewing angles, however, these formed objects usually suffer from object distortion. To prevent the distortion, some related work limits viewing angles and some do not use lenses and mirrors to form volumetric objects. In this study, we modeled the distortion and pre-warped the objects, so that they would form not-distorted. Finally, we confirmed that our approach had the potential to correct the distortion.

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